LESSON 3.3a

Introduction to Completing the Square

Today you will:

- Get more practice solving quadratic equations using square roots.
- Learn/review what a ***perfect square trinomial*** is.
- Learn what ***completing the square*** is and how to use it.
- Solve quadratic equations $x^2 + bx + c = 0$ $(a = 1)$ by completing the square

Core Vocabulary:

- Perfect square trinomial

- Completing the square, p. 112

What is a “perfect square?”

- A number that can be expressed as the product of two equal integers.
- …a number that is the square of an integer.

Can you think of some examples?

- 4 (2 * 2)
- 9 (3 * 3)
- 121 (11 * 11)
- 529 (23 * 23)

What is a "trinomial?"

- An expression with three terms connected with plus and/or minus.

Can you think of some examples?

- $x^2 + 3x + 2$
- $5x^2 - 2x + 12$

(these are both quadratics … note that a trinomial does not need to be a quadratic)

What is a "perfect square trinomial?"

- A trinomial that can be factored into a binomial multiplied by itself.

(what is a binomial? … an expression with two terms connected with plus and/or minus)

Examples*:*

- $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$
- $x^2 - 6x + 9 = (x - 3)(x - 3) = (x - 3)^2$

Solve $\boldsymbol{(x-8)^2=100}$

$$(x-8)^2=100$$

$$\sqrt{(x-8)^2}=\sqrt{100}$$

$$x-8=\pm 10$$

$$x=8\pm 10$$

ANOTHER WAY

You can also solve the equation by writing it in standard form as $x^2 - 16x - 36 = 0$ and factoring.

Solve $x^2 - 16x + 64 = 100$ using square roots.

SOLUTION

$x^2 - 16x + 64 = 100$	Write the equation.
$(x - 8)^2 = 100$	Write the left side as a binomial squared.
$x - 8 = \pm 10$	Take square root of each side.
$x = 8 \pm 10$	Add 8 to each side.

So, the solutions are $x = 8 + 10 = 18$ and $x = 8 - 10 = -2$.

What would I have to add to $x^2 + 4x$ to make it a perfect square trinomial?

In other words given $x^2 + 4x + c$, what would c have to be to make it a perfect square trinomial?

Let's think about this:

- In $x^2 + bx + c$ remember we have said that $b = p + q$ and $c = p \cdot q$
- …here we are wanting to make a perfect square trinomial…
- …what does that mean/say about $p \; and \; q$?

    - Well, c will need to be a perfect square … which means $c = p^2$ … in other words $p = q$
    - If $c = p \cdot p$ (and $p = q$) what can we say about b?

    - $b = p + q$ and if $p = q$ then $b = p + p = 2p$

In this problem $b = 4$ which means $4 = 2p$ so $p = 2.$ Take b, divide it in two, then multiply it by itself…
So $c = 2 \cdot 2 = 4$

To make $x^2 + bx + c$ a perfect square $c = \left(\frac{b}{2}\right)^2$

COMPLETING THE SQUARE

To complete the square for the expression $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)\left(x + \frac{b}{2}\right) = \left(x + \frac{b}{2}\right)^2$$

Example: Complete the Square for $x^2 + 18x$ (make it a perfect square trinomial)

$b = 18$

$c = \left(\frac{b}{2}\right)^2 = \left(\frac{18}{2}\right)^2 = 9^2 = 81$

So the answer is: $x^2 + 18x + 81$

Find the value of c that makes $x^2 + 14x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

SOLUTION

Step 1 Find half the coefficient of x. $\frac{14}{2} = 7$

Step 2 Square the result of Step 1. $7^2 = 49$

Step 3 Replace c with the result of Step 2. $x^2 + 14x + 49$

The expression $x^2 + 14x + c$ is a perfect square trinomial when $c = 49$. Then $x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$.

Using COMPLETING THE SQUARE to solve a quadratic

Solve $x^2-18x+5=0$:

$$x^2-18x+5=0$$
$$\underline{\qquad\qquad -5 \quad -5}$$
$$x^2-18x \quad = -5$$

First move the +5 to the other side…

…now complete the square

$$b=18$$
$$c=\left(\frac{b}{2}\right)^2=\left(\frac{-18}{2}\right)^2=(-9)^2=81$$

$$x^2-18x+\mathbf{81}=-5+\mathbf{81}$$

★★★ **If you add to one side, you *MUST* add to the other** ★★★

$$x^2-18x+81=76$$

…now factor and solve…

$$(x-9)(x-9)=76$$
$$(x-9)^2=76$$
$$\sqrt{(x-9)^2}=\pm\sqrt{76}$$
$$x-9=\pm\sqrt{4\cdot 19}$$
$$x=9\pm 2\sqrt{19}$$

LOOKING FOR STRUCTURE

Notice you cannot solve the equation by factoring because $x^2 - 10x + 7$ is not factorable as a product of binomials.

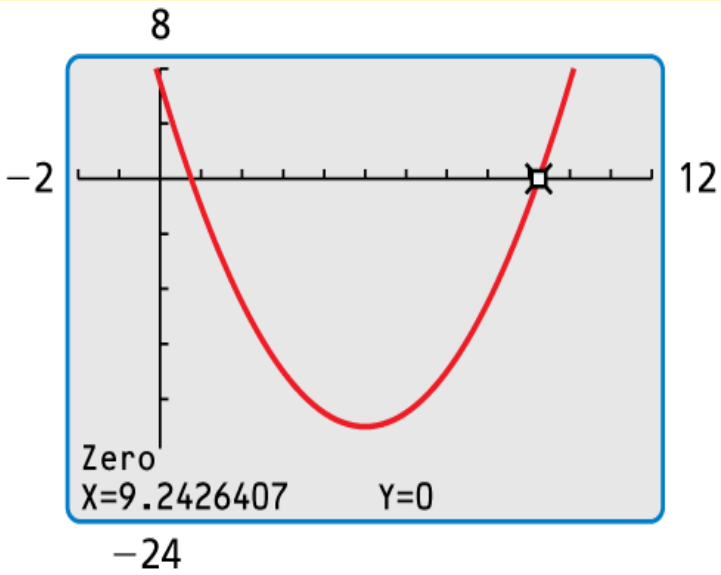
Solve $x^2 - 10x + 7 = 0$ by completing the square.

SOLUTION

$x^2 - 10x + 7 = 0$	Write the equation.
$x^2 - 10x = -7$	Write left side in the form $x^2 + bx$.
$x^2 - 10x + 25 = -7 + 25$	Add $\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = 25$ to each side.
$(x - 5)^2 = 18$	Write left side as a binomial squared.
$x - 5 = \pm\sqrt{18}$	Take square root of each side.
$x = 5 \pm\sqrt{18}$	Add 5 to each side.
$x = 5 \pm 3\sqrt{2}$	Simplify radical.

The solutions are $x = 5 + 3\sqrt{2}$ and $x = 5 - 3\sqrt{2}$. You can check this by graphing $y = x^2 - 10x + 7$. The x-intercepts are about $9.24 \approx 5 + 3\sqrt{2}$ and $0.76 \approx 5 - 3\sqrt{2}$.

Check



Homework

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